Force Control for Planar Spring-Mass Running

Devin Koepl and Jonathan Hurst

***Abstract*— In this paper, we present a novel control strategy for spring-mass running gaits which is robust to disturbances, while still utilizing the passive dynamic behavior of the mechanical model for energy economy. Our strategy combines two ideas: a flight phase strategy, which commands a hip angle trajectory prior to touchdown, and a stance phase strategy, which treats the spring-mass system as a force-controlled actuator and commands forces according to an ideal model of the passive dynamics. This combined strategy is self-stable for changes in ground height or ground impedance, and thus does not require an accurate ground model. Our strategy is promising for robotics applications, because there is a clear distinction between the passive dynamic behavior and the active controller, it does not require sensing of the environment, and it is based on a sound theoretical background that is compatible with existing high-level controllers.**

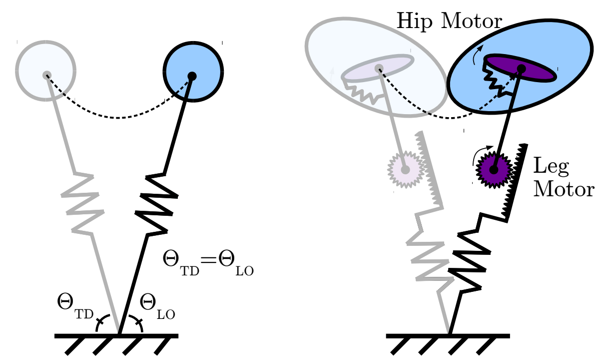
I. INTRODUCTION

Walking and Running robots, in general, have significant ground to cover before they can approach the abilities of animals. Walking and running animals are able to attenuate significant disturbances, such as uneven ground, while maintaining excellent energy economy. Existing passive walkers, such as the Cornell Walker, are capable of energy economy similar to animals, but will fall in the presence of small disturbances [1]. Robots that rely primarily on active control, such as Boston Dynamics' "BigDog," can demonstrate impressive robustness to disturbances at the expense of energy economy [2]. Our goal is to create robots that combine the robustness to disturbances of actively controlled machines with the energy economy of a passive dynamic walker.

In this paper, we present an actuated spring-mass model that is suitable for implementation as a real system, shown in Fig. 1(b), and an associated control strategy for planar running. The control strategy works in conjunction with our model to utilize the passive dynamics where possible for energy economy, and to add or remove energy only when necessary via actuation. We show in simulation that the combined model and controller is energetically conservative like the completely passive spring-mass model during steady-state running, but is self-stable in the presence of disturbances in the ground height or impedance. In other words, our model and controller combine the benefits of passive dynamics and active control, producing an efficient and robust running gait.

The control strategy in this paper combines two concepts: control of the leg angle position during flight, similar to that described by Seyfarth et al., and active force control in the leg length direction during the stance phase, described in our earlier publication for vertical hopping [3], [4].

This work was supported by the school of Mechanical, Industrial and Manufacturing Engineering at Oregon State University. The authors can be contacted by email at:
koepld@onid.orst.edu



(a) The passive spring loaded inverted pendulum (SLIP) model.
(b) Our actuated spring-mass model.

Fig. 1. Starting with the passive SLIP model, we add hip and leg motors for active control, and body moment of inertia.

Our leg angle controller is based on maintaining a symmetrical stance phase, where the velocity vector of the center of mass at liftoff will be a perfect mirror of the velocity vector at touchdown; the horizontal component will be identical, and the vertical component will be equal and opposite, as shown in Fig. 1(a). A symmetrical stance phase leads to an equilibrium gait, because each stance phase is identical to the last if there are no outside disturbances. For a given spring-mass model, with a particular center-of-mass velocity vector at touchdown, there is a particular leg angle at touchdown that will result in a symmetrical stance phase. We calculate this leg angle for each instant of time during the flight phase, as the velocity vector changes, such that the spring-mass model will have a symmetrical stance phase no matter when its foot makes contact with the ground.

Controlling the leg angle for a symmetrical stance phase will ensure an energetically optimal, stable gait on unpredictable, uneven terrain. However, changes in ground impedance, such as changes in ground stiffness or damping, will destabilize the gait. To handle such disturbances, we implement stance phase force control. The actuators control the deflection of the leg spring as a function of time, and thus the toe force as a function of time, to match the ideal force profiles of the undisturbed passive spring-mass system. In the absence of disturbances the result is that the motor holds its position, and the spring does all the work. By maintaining a specific force profile, the leg appears to automatically adjust its stiffness to accommodate changes in ground stiffness, to adjust its zero force leg length to attenuate changes in ground surface height, and to add energy to accommodate increased

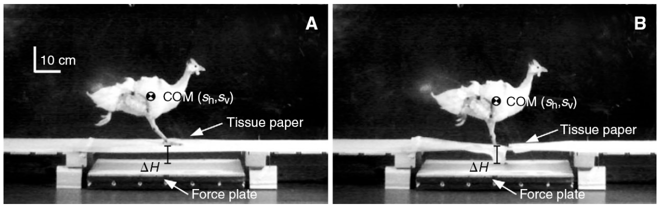


Fig. 2. Motivation comes from the economy and disturbance rejection ability of animals such as the guinea fowl. The guinea fowl is able to accommodate for the unexpected decrease in ground surface without a significant change to its steady-state center of mass motion.

ground damping.

Our simulation results show that our control strategy can make the spring-mass model, hopping in the vertical plane, robust to ground disturbances with limited sensory input. Our self-stability strategy is based only on the spring deflections, center of mass velocity, motor, and body angles, and not on any external sensing, which makes it practical for legged robots, such as ATRIAS, a bipedal robot currently under development in our lab, shown in Figures 8(a) and 8(b), that has incomplete knowledge of the world. While the passive dynamics of the system attenuate very high-frequency disturbances, the force controller focuses on the middle-frequency disturbances, leaving any high-level gait choices or stride-to-stride control to a higher-level control system. This novel concept of combining force control and a spring-mass model is convenient, easy to implement on a real system with dynamic and sensing limitations, and effective.

II. BACKGROUND

Because our goal is to build robots that can match the performance, economy, and robustness of animal running, our models incorporate passive dynamics similar to those observed in animals. Spring-mass models provide a good approximation for animal running, we therefore begin with a simple model consisting of a mass bouncing on a spring, as shown in Fig. 1(a) [5].

Humans and animals make excellent use of passive dynamics, but also use active control to compensate for disturbances. For example, guinea fowl are able to accommodate a drop in ground height by rapidly extending their leg into an unexpected disturbance, as shown in Fig. 2, resulting in only slight deviation from their undisturbed gait [6]. Furthermore, biomechanics studies suggest that humans and animals adjust their leg stiffness during hopping, walking, and running to accommodate changes in ground stiffness and speed [7]. These types of active responses to ground disturbances are important on physical systems, where deviations from the undisturbed gait can lead to a loss of stability, falls, or springs exceeding their maximum deflection, potentially causing damage. For example, galloping horses are already near peak force on tendons and bones, so remaining below force limits is an important consideration, or small ground disturbances could result in injury or damage [8].

The simple spring-mass model is capable of some passive stability, but without careful control of the leg angle at touchdown it tends to become unstable and fall [9]. A simple leg angle controller based on tuplets of natural frequency, zero-force leg length, apex hop height, and horizontal velocity may yield stable hopping gaits. Existing methods for selecting leg touchdown angles have included hand-tuned gain based controllers and constant leg retraction velocity control [3]. However, these methods require tuning, and are subject to controller optimality.

A more reliable method of selecting a leg touchdown angle for SLIP model running, presented by Ernst et al., prevents falls by ensuring a center of mass trajectory during stance that is symmetrical about midstance [10]. We call this type of gait an equilibrium gait, because every stride is the same as the last, and in the interest of brevity, will henceforth refer to Ernst et al.'s method of selecting the leg touchdown angle as the Ernst-Geyer-Blickhan (EGB) method.

Equilibrium gaits are desirable for applications such as ours, where a consistent center of mass trajectory and toe force profile are desired. We therefore adopt the EGB method as a baseline for comparison and use it on our force-controlled model for setting the flight phase leg angle to the equilibrium gait leg touchdown angle. Controlling the leg angle in this way requires a method for finding the equilibrium gait leg touchdown angle, but since the stance phase dynamics of the spring-mass model hopping with non-zero horizontal velocity in the vertical plane are non-integrable, analytically computing this equilibrium gait leg touchdown angle is not possible [11]. This limits us to numerical solutions or analytical approximations such as those presented by Geyer et al. [12]. For the purposes of this paper, we generate a three input look up table, but a trained neural network was also considered, and other approximations could yield similar results.

Springs clearly help running gaits by storing and releasing energy, but they are also useful for force control, which can improve the robustness of running gaits. The MIT Series Elastic Actuator (SEA) measures and controls the deflection of its spring, which corresponds to the force applied by the actuator [13]. As an added benefit, the series spring filters impulsive forces, improving the SEA's robustness to shock loads [14]. The performance of force-controlled actuators, such as the SEA, has been explored, and some task-specific criteria for selecting actuator dynamics have been identified, but these investigations are not generally extended to robot walking and running [15]. However, force control using the deflection of series springs has been successfully implemented on legged robots such as Boston Dynamics's walking and running quadruped, "BigDog", and the MIT Leg Lab's walking biped, "Spring Flamingo" [16], [17]. These robots use springs in much the same way as the SEA, as a force sensor and mechanical filter, but not for energy storage. When correctly applied, this approach can result in impressive performance, but at the cost of high energy consumption.

III. MODEL

Starting with the simple spring-mass model, shown in Fig. 1(a), we add hip and leg actuation as well as body moment of inertia to arrive at a realistic model for robot running, as shown in Fig. 1(b). The actuators include a motor with a torque limit and rotor inertia. We chose to omit leg mass from our model, to keep the system as simple as possible. For ATRIAS, shown in Figures 8(a) and 8(b), our eventual target robot for this control method, toe mass composes less than one percent of total robot mass. The leg actuator makes use of the existing leg spring, while we add a second rotational spring to the model for the hip actuator. The hip actuator sets the leg angle during flight and maintains zero moment about the hip during stance, such that the force-controlled model behaves like the passive model during undisturbed hopping.

We use kinetic equations of motion to simulate our force-controlled model hopping in the vertical plane. Although developing these non-linear equations is straightforward for both the standard spring-mass model and the force-controlled model, finding a closed-form solution for the trajectories of either is impossible [11]. However, we are able to generate approximate numerical solutions using ordinary differential equation solvers.

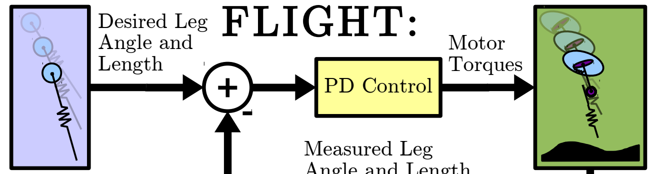
IV. CONTROL STRATEGY

Our controller attempts to match the force-controlled model's toe force profile to that of an equivalent undisturbed spring-mass model, such that the center of mass movement is roughly the same for both.

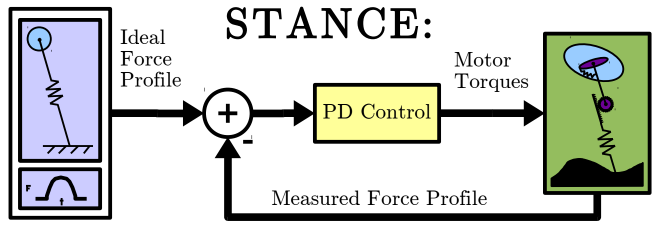
During flight, we use PD position control of the hip motor and the EGB method to set a leg angle trajectory, such that the model touches down in an equilibrium gait regardless of when the toe contacts the ground. We use a numerical solution for the SLIP model along with a gradient descent algorithm to generate a lookup table for the equilibrium gait leg angle, in much the same way as presented by Ernst et al., but we also consider the zero-force leg length as an input to our leg angle lookup table, since it varies in response to ground surface disturbances [10].

In stance, we use PD torque control of the hip actuator to approximate an ideal hinge, such that the force-controlled model behaves like a SLIP model in stance with a point-mass body. The hip motor must track the motion of the leg in stance to maintain zero deflection of the hip spring; this task is equivalent to maintaining constant force against a moving load, a task which has been approached analytically in previous works [15]. Although optimal performance for a force control task would require very low motor inertia and spring stiffness, we use realistic values from the design of a legged robot we are currently constructing, shown in Fig. 8(a). The limitations imposed by these realistic passive dynamics are represented in our results.

The leg actuator attenuates ground disturbances by controlling the force in the leg spring. The leg spring stiffness is tuned to the natural frequency of our desired spring-mass hopper, so energy will be stored in the spring during the first part of stance, and then recovered as the body mass



(a) In flight we set the leg angle using the Ernst-Geyer-Blickhan method, and the zero force leg length such that it remains equal to that of the equivalent passive model, unless there is a change in ground height.



(b) During stance we command the toe force profile of the passive model hopping on flat rigid ground.

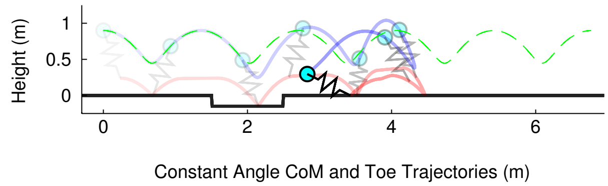
Fig. 3. Our approach switches between flight and stance controllers according to whether force is present in the model's springs.

accelerates towards liftoff. In the ideal scenario, the leg motor does no work, the hip motor is only responsible for moving its own inertia and does no work on the environment, and all of the model's behavior is expressed by the passive dynamics of the system as it bounces forward. In the presence of disturbances, critically damped PD force control of the leg motor generates torques such that a toe force profile can be followed provided that the motor's torque limits are not exceeded. As a result of attempting to control the leg force, if the toe is not yet in contact with the ground when the robot expects to have made contact, the leg will rapidly extend until it makes contact with the ground, and the desired force can be applied.

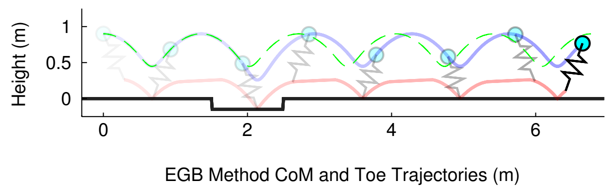
Our active control strategy maintains the passive dynamics of the equivalent spring-mass system, and intervenes with the passive dynamics of the simple model only to accommodate ground disturbances. When our model encounters an unexpected change in ground height or stiffness, the leg extends or retracts such that the toe forces match those of the undisturbed passive dynamics. During undisturbed hopping our simulation behaves like a simple spring-mass model with minimal interference from active controllers. Aside from setting the leg touchdown angle, active control does work only when a ground disturbance is encountered.

V. SIMULATIONS

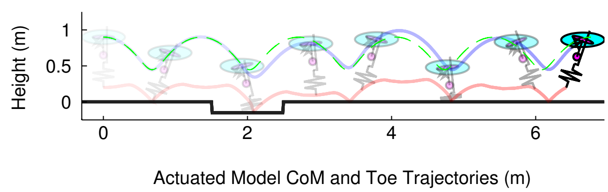
We compare, in simulation, three models: a passive spring-mass model with constant leg angle at touchdown, a spring-mass model that adjusts its leg angle in flight according to the EGB method, and our force-controlled model that combines the EGB method with force control. The three models are subjected to changes in ground height and ground stiffness. To better demonstrate the feasibility of disturbance rejection on our model in simulation, we choose somewhat arbitrary, but realistic values for a moderately-sized robot using a commercially available motor, such as our ATRIAS



(a) The passive spring-mass model with constant leg touchdown angle encountering a decrease in ground surface height. The model falls shortly after the disturbance.



(b) A standard spring-mass model using the EGB method to select the leg touchdown angle encountering a decrease in ground surface height. Although the model does not fall, its center of mass trajectory is affected by the disturbance.



(c) The force-controlled model encountering a decrease in ground surface height. The center of mass trajectory is affected, but the deviation that occurs is a result of motor limitations.

Fig. 4. Center of mass and toe trajectories for the standard spring-mass models and the force-controlled model encountering an unexpected decrease in ground surface. The dashed line represents the center of mass trajectory of the passive undisturbed model.

monopod, shown in Fig. 8(a).

The unactuated spring-mass models are not subjected to any of the physical limitations that we impose on our force-controlled model. They are able to instantaneously set their leg angle, their hip behaves like an ideal hinge during stance, and they do not have motors that can hit their torque limit, accumulate angular momentum, or be backdriven.

We expect ground disturbances to result in permanent shifts in hopping phase and height for the standard spring-mass models, if not a loss of stability and falls. However, we expect our force-controlled model to accommodate ground disturbances and to closely follow the toe force profiles and center of mass trajectory of the undisturbed system.

A. *Ground Height Disturbance*

The first type of disturbance we investigate is a decrease in ground height. There are no "sensors" that allow the model to change its control strategy, and it has no forewarning of the change in ground height. After its first hop on the flat rigid surface, the model takes its second hop onto the lower ground surface, and the ground surface then returns to its original height for a third hop. We choose a ground height disturbance of sufficient magnitude to cause the passive model with constant leg touchdown angle to become unstable and fall.

For the standard spring-mass models, variations in ground height affect the toe force profile and center of mass trajectory, as shown in Fig. 4(a), 4(b), and 5. When these models encounter a drop in ground height, they touchdown with greater velocity and the toe force profile exceeds that of the undisturbed model. The spring-mass model with constant leg touchdown angle becomes unstable in the example we present, and falls soon after the disturbance, as shown in Fig. 4(a), but the spring-mass model using the EGB method hops through the disturbance without losing stability. The decrease in ground height disturbance causes the model with constant leg touchdown angle to lose horizontal velocity, making its leg touchdown angle too shallow, whereas the model using the EGB method maintains a constant horizontal velocity.

In contrast to the passive spring-mass models, the toe force profile of our force-controlled model is roughly maintained despite changes in ground height, such that the peak force does not increase from the undisturbed gait. The active control system extends the leg as quickly as possible, given the leg motor torque limit, at the time of expected toe impact. When toe impact occurs, the leg actuator begins to track the force profile of the equivalent passive system hopping on flat rigid ground. The result is a small phase shift in the center of mass trajectory and toe force profile, as shown in Fig. 5, that is caused by the leg motor torque limit and inertia. These physical limitations and the lack of sensing of the environment makes some deviation inevitable, but we note that we can greatly reduce the deviation using the approach we describe.

B. *Ground Stiffness Disturbance*

The second type of ground disturbance we investigate is a decrease in ground stiffness. For this experiment, the ground unexpectedly changes from being perfectly rigid to behaving like an ideal spring in all directions. As a model touches down, the ground depresses proportionately to the model's toe force. The net result is that the model experiences a

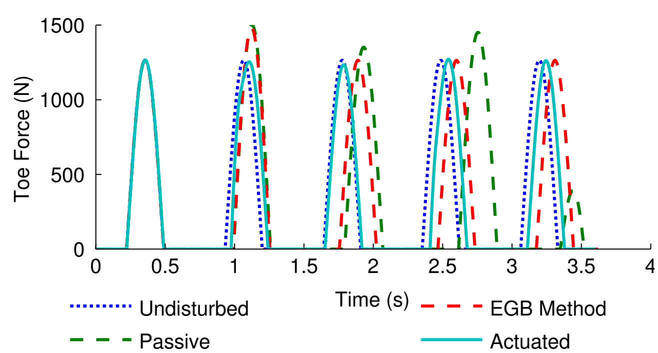


Fig. 5. Toe force profiles for the passive undisturbed, passive disturbed, passive with Ernst-Geyer-Blickhan leg touchdown angle, and force-controlled models encountering a decrease in ground surface height. Our force-controlled model is nearly able to maintain the toe force profile of the undisturbed model, and most importantly, is able to limit the peak forces. In contrast, both passive models show a significant change in the force profile, and an increase in peak forces.

leg spring stiffness equal to the series combination of its leg spring with the ground spring, and the hip stiffness is similarly affected.

For the standard spring-mass models, variations in ground stiffness lead to immediate falls or, at best, a large change in center of mass trajectory and toe force profile, as shown in Fig. 7(a), 7(b), and 6. When these models touchdown on a non-rigid surface, their apparent leg stiffness becomes the series combination of their leg spring and the ground stiffness, and their leg touchdown angle, which would have produced an equilibrium gait on rigid ground, is too steep for the decrease in stiffness. Even for small decreases in ground stiffness, the standard spring-mass model with constant leg touchdown angle becomes unstable and falls. The standard spring-mass model with EGB leg touchdown angle can avoid falls due to ground stiffness perturbations by adjusting its leg touchdown angle to compensate for the higher horizontal velocity and lower hopping height that result from the disturbance. However, the EGB method will fail if a disturbance is severe enough that liftoff does not occur or an equilibrium gait touchdown angle cannot be found, as shown in Fig. 7(b).

In contrast, the force-controlled model maintains the toe force profile of the equivalent undisturbed spring-mass model despite changes in ground stiffness. Controlling the toe force profile results in the leg extending into the soft ground during the first half of stance and retracting during the second half, such that the toe force profile and apparent leg stiffness are roughly the same as for the undisturbed model hopping on flat rigid ground, as shown in Fig. 7(c). As with the ground height disturbance, there is a slight deviation away from the undisturbed center of mass trajectory that results from motor limitations. This is because at the instant of touchdown, infinite motor torque is required to instantaneously give the leg motor the angular velocity necessary to perfectly match the toe force profile of the undisturbed system, but we limit the motor torque to that of a commercially available motor.

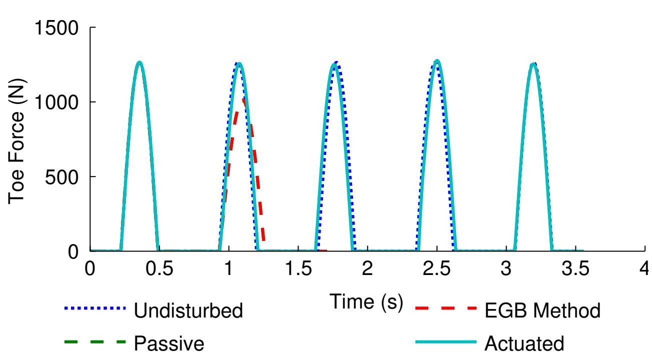
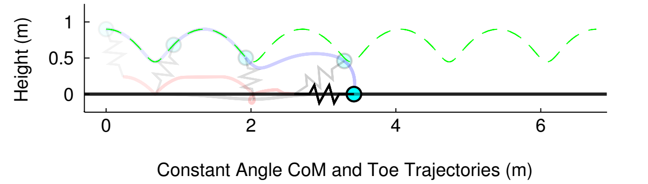
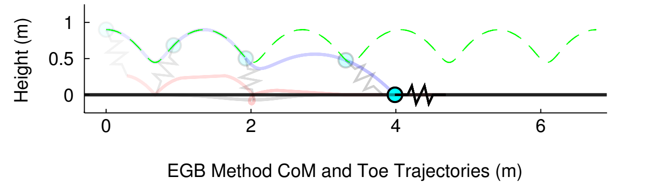


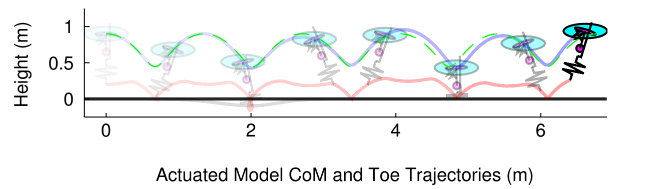
Fig. 6. Toe force profiles for the passive, passive with Ernst-Geyer-Blickhan leg touchdown angle, and force-controlled models compared to the undisturbed passive model encountering a decrease in ground stiffness. The toe force profiles of the passive models encountering the disturbance exceed and deviate away from the toe force profile of the force-controlled model, which is able to maintain the toe force profile of the undisturbed passive model, aside from a small phase shift caused by motor limitations.



(a) The passive spring-mass model with constant leg touchdown angle encountering a decrease in ground stiffness. The model falls in the presence of the disturbance.



(b) The passive spring-mass model using the EGB method to set the leg touchdown angle encountering a decrease in ground stiffness. The model bounces out of the disturbance at such a low trajectory that there is no leg touchdown angle that will prevent a fall.



(c) The force-controlled model is able to attenuate the unexpected decrease in ground stiffness by maintaining the ground force profile of the ideal model. The center of mass trajectory is scarcely affected by the unexpected disturbance.

Fig. 7. Simulation results for our robot model and spring-mass models encountering an unexpected decrease in ground stiffness. The dashed line represents the center of mass trajectory of the passive undisturbed model.

VI. CONCLUSIONS AND FUTURE WORK

We have shown in simulation that active force control combined with careful control of the leg touchdown angle yields good disturbance rejection for spring-mass model running, while requiring little sensory feedback and minimal active control. An untethered robot using our control strategy would require only joint position sensors, and an inertial measurement unit for sensing the center of mass velocity during flight. During steady state hopping, the model predominantly relies on its passive dynamics as it hops forward, maintaining much of the energy economy of the entirely passive system. In the presence of disturbances, we use active motor control to approximate the toe force profile of the passive spring-mass model on a flat rigid surface. The deviation that exists is due to motor limitations, and diminishes with greater control authority. Because the toe force profiles are close, the model's center of mass movement follows that of the undisturbed ideal passive system.

The simulation illustrates some system limitations caused by the motors' torque limit and inertia. When the force-controlled model encounters a drop in ground height, there is a delay between when toe touchdown is expected to occur, and when it actually does. The delay depends on the magnitude of the disturbance and motor limitations. During this delay, the motor gains angular momentum, causing a slight asymmetry in the stance of the force-controlled model. The simulated motor has a realistic inertia that can only be accelerated and decelerated as quickly as the motor's torque limit allows. The error in the toe force profile can be minimized by tuning control constants, but the controller does not know the position of the ground or any other information about the world, so we presume that it cannot be eliminated without additional sensory input. Despite this sensor limitation, our force controlled model outperforms the idealized un-implementable SLIP model using the EGB method to set its leg touchdown angle. It is able to perform well, with only a small discrepancy between the actual and desired toe force profiles, even in the presence of a substantial change in ground height.

Sensory and physical limitations affect the ability of the model to remain in an equilibrium gait. The performance of the in-flight leg touchdown angle controller is dependent on the accuracy of force and angle measurements during stance, the accuracy of the approximation method, the angular momentum of the leg motor at touchdown, and the control authority of the the hip motor. On a physical system with sensory, computing, and physical limitations, this dependency is unavoidable, and while such limitations could easily be lifted from our simulation, we chose to include them to better illustrate the feasibility of taking our force control approach on a physical system.

The long-term goal of this work is to build a model and bio-inspired biped capable of robust walking and running gaits. Previously we showed how force control could be used to add disturbance rejection to a vertically hopping force-controlled spring-mass model [4]. We have now shown how this idea can be extended to the vertical plane in simulation on a model with similar limitations to a physical system we are building, shown in Fig. 8(a). We are now working to demonstrate our concept of adding force control to the spring-mass model on physical systems, including a single degree of freedom benchtop actuator, a two degree of freedom monopod, and eventually on a tether-free biped. Our biped will maintain as much of the energy economy of the equivalent passive system as possible by making excellent use of its passive dynamics, while limiting the need for sensory feedback and active control. With these real-world devices, we hope to approach the performance of animal walking and running.

(a) A monopod version of ATRIAS, with hip and leg series elasticity.

(b) Bipedal ATRIAS, including on-board power and computing.

Fig. 8. We are currently working toward validating our simulation results on purpose-built machines of our own design and construction.

ACKNOWLEDGEMENTS

We wish to acknowledge Christian Hubicki's assistance proofreading this paper, Jesse Grime's rendering of Figures 8(a) and 8(b), and the support of the Human Frontier Science Program.

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