

## Series Elastic Actuators

the date of receipt and acceptance should be inserted later

### Author Info

Prof. Jonathan Hurst  
Department of Mechanical, Industrial and Manufacturing Engineering  
Oregon State University  
Corvallis, OR 97331  
United States of America  
jonathan.hurst@oregonstate.edu

Kevin Green  
Department of Mechanical, Industrial and Manufacturing Engineering  
Oregon State University  
Corvallis, OR 97331  
United States of America  
greenkev@oregonstate.edu

### Definition

A series elastic actuator is an actuation unit in which the motion source is connected to the output through an intentional, engineered elastic element which can allow for high fidelity force control, impact tolerance and energy storage.

### Introduction

This article describes the practice of utilizing elasticity in series with a prime mover, such as an electric motor or hydraulic cylinder, for actuation tasks. The term “series elastic actuator” commonly describes an integrated actuation unit that contains a spring whose deflection is measured for use in a closed loop force controller. However, this article focuses on the more general case of actuators that

---

utilize one or more series elastic elements to intentionally improve the dynamic and control properties of the system. This may be a single standalone unit, or it may be an integrated multi-degree of freedom system. Further, the performance measure (and utility of the series compliance) will depend entirely on the specific task required of the actuation system.

All actuators have limitations, and series elasticity is only ever added to accommodate these limitations. Some examples of these limiting factors are inertia, damping, backlash and torque limits in a geared electric motor, or fluid inertia, stiction and flow restrictions in a hydraulic cylinder. Most robots are designed mechanically to have as rigid a connection between a prime mover and the mechanism as possible, with the desired behaviors implemented in software control. However, for many tasks, the actuator dynamics can limit the behaviors an actuator can perform, regardless of the control policy (Braun et al. 2012). Adding series elasticity can change the behaviors a given actuator is best suited to achieve; note that it does not improve performance for all tasks (Hurst et al. 2004). One must consider both the specific actuator dynamics as well as the tasks to be performed. In the design of a series elastic actuator, many design choices are engineering trade-offs, helping some tasks while limiting others.

Actuator series elasticity is particularly useful for physical interaction with uncertainty. In robot arms, for example, adding tuned and sensed physical compliance can improve their ability to behave stably when compliance is an important behavioral trait (Bicchi et al. 2005). In assistive prosthetics, careful control of applied forces is important for user comfort. Implementing force control is dramatically improved by using series elasticity to sense forces through deflections and to moderate force error from environmental interaction (De Schutter 1987). For legged locomotion, series elasticity can be used to store and release energy of a cyclic gait, creating a specific oscillatory energy cycle. In addition, the spring can be used to amplify the power output of the prime mover (Roberts 2002). In each of these cases elasticity must be tuned to the specific task to improve performance rather than degrade it.

## Overview: Physics are the Bottleneck

**Actuator Dynamics limit possible behaviors.** Actuators are not ideal force, torque or motion sources. They are complex systems that have intrinsic dynamics, and these dynamics must be considered during design of the mechanism and controller. We will use electric motors for most examples in this paper, as they are most common in robotic systems; and electric motors, with a transmission, all have reflected inertia, which is the effect of accelerating actuator components to high speeds through a transmission. This reflected inertia combined with torque limits on the motor will limit the possible acceleration of the actuator. For geared electric motors, the reflected inertia can be sizeable, sometimes comparable to the inertia of the entire robot mass (Hurst et al. 2004). Even with perfect sensing and control, the intrinsic physics of the actuator exists; all actuators have their own internal dynamics and associated limitations.

If the output of an actuator with large reflected inertia is subjected to a collision, there will be a large impulse, high forces in the gearing, high frequency oscillations and energy losses. During a collision, series elasticity isolates the large

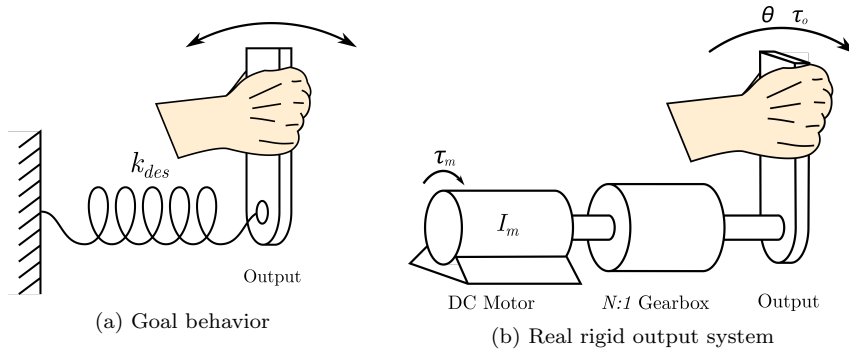


Fig. 1: A gearmotor will fail to emulate a spring as the motion speeds up. This is due to the reflected inertia and torque limits that are a property to the motor itself, independent of the control algorithm.

reflected inertia of the motor from the output. This serves at least two purposes: it reduces the magnitude of the impulse, protecting the system from high peak forces; and it lengthens the amount of time of the impulse, allowing the control system time to accelerate the inertia of the actuator and take action.

For a concrete example of actuator limitations, consider an electric gear motor employed to behave like an ideal torsion spring as a part of a haptic system (Fig. 1). To highlight the effect of motor dynamics, assume an omniscient controller which knows the exact state of the system and can choose the perfect torque to apply to the rotor such that the system emulates a spring. However, it is constrained to real-world physics, including torque limits of the motor and mechanism. When a user moves the output link back and forth, they should feel a reaction torque that exactly mimics a torsion spring. For this to happen, the motor has to apply torque to do two things: emulate the spring and accelerate the motor's inertia. As the user moves the output faster and faster, the torque required to accelerate the inertia becomes larger and larger. At a large enough frequency and amplitude of motion the torque required will surpass the maximum torque limit; and, given common values for reflected inertia of geared motors, this frequency and amplitude is quite low and easily reached. At this point the user will feel some of the motor's inertia, even with an omniscient controller.

To describe the phenomenon more precisely, consider the same system's dynamics,

$$N^2 I_m \ddot{\theta} = N \tau_m + \tau_o,$$

where  $N$  is the speed reduction of the gearing,  $I_m$  is the inertia of the rotor,  $\theta$  is the angle of the output,  $\tau_m$  is the torque applied by the motor on its rotor, and  $\tau_o$  is the torque applied by the external load. The desired behavior for this system is to have the torque on the output be  $\tau_o = k_{des} \theta$ . Assume the operator moves the output in a sinusoidal trajectory defined by  $\theta(t) = A \sin(\omega t)$  regardless of the required torque. If the control system perfectly emulates a passive spring, then the operator will need to apply a torque of  $\tau_o(t) = k_{des} A \sin(\omega t)$  to the system. It follows that the motor will need to apply a torque of

$$\tau_m(t) = -A(N I \omega^2 + k_{des}/N) \sin(\omega t)$$

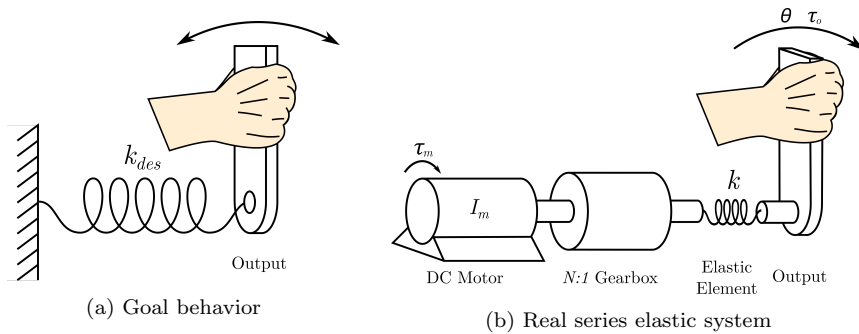


Fig. 2: A gearmotor with series elasticity can perform a force control task with much higher performance compared to the rigid case from Fig. 1.

to ensure the user only feels the spring force. If this required  $\tau_m(t)$  exceeds the torque limit, then the motor cannot behave like the desired spring in this application. These limits are often surprisingly low, due to the large gear reductions used in common robot systems, causing a large reflected inertia ( $N^2 I_m$ ) when combined with even very small rotor inertia.

**Series elasticity can help overcome some of these limitations, depending on the behaviors you want.**

Consider the same example as before with an actuator mimicking a torsion spring, but now an elastic element is inserted between the gearbox and the output (Fig. 2). Once again, assume an omniscient controller working within constraints of actuator torque limits. If the elastic element is perfectly matched to the task, i.e. it has the same stiffness and no damping, then the motor only needs to provide the correct torque to hold the output of the gearbox stationary. In this case the system can behave like the spring at all frequencies as long as the user torque does not exceed the motor torque limit. In other words, it has infinite bandwidth for this particular task.

If the elastic element is not perfectly matched to the task, performance may still improve. Consider the case where the elastic element is 50% stiffer than the desired behavior,  $k = 1.5k_{des}$ . Unlike in the perfectly matched case, the motor now has to move to apply the correct force to the output. However, it has to move less because much of the movement of the output is accounted for by the deformation of the elastic element.

To see this improvement, consider the new system with the same motion and reaction torque as the previous example:

$$\theta(t) = A \sin(\omega t) \quad \text{and} \quad \tau_o(t) = k_{des} A \sin(\omega t).$$

By torque balance at the spring,

$$\frac{3}{2} k_{des} (\theta(t) - \theta_m(t)) = k_{des} A \sin(\omega t),$$

one can find that the required motor trajectory is  $\theta_m(t) = \frac{1}{3} A \sin(\omega t)$ . The torque from the motor must both accelerate the motor inertia and apply the reaction

force to the spring. The total motor torque required is now

$$\tau_m(t) = -A\left(\frac{1}{3}NI\omega^2 + k_{des}/N\right)\sin(\omega t).$$

The motion required of the motor is now only one third of what it was in the rigid case, which reduces the total torque required.

**Actuators are not linear systems and should not be analyzed like one.**

One of the most common metrics to describe actuator capabilities is bandwidth. In the case of actuators, bandwidth is used to mean the range of frequencies at which an actuator can accurately perform a specified motion (such as a sine wave). However, bandwidth carries the assumption that a system is linear, and performance is independent of the magnitude of motion. For all actuators, many of the important limits of performance are results of nonlinear effects, mainly torque limits. This nonlinearity means that bandwidth shown through bode plots does not capture some of the most important performance limits of any actuator, whether series elastic or not. Frequency plots at a defined amplitude of motion are useful, but it is important to remember that these will change dramatically with amplitude of motion.

**Performance is task-dependent: knowing the task is necessary for actuator design.** Consider two extreme situations when describing the performance of a force control actuator: applying constant force against a moving object (for example, a holding a cup of coffee in your hand while in a car on a bumpy road), and applying varying force on a stationary object (such as holding a perfect position even while being pushed). In the case of applying a constant force to a moving object, the limit to *perfect* performance is defined by the reflected inertia of the motor/transmission unit and the torque limits, not by the elasticity, because a constant force requires a constant deflection of the elastic element. The only way to maintain a constant deflection is to have the inertial load of the motor *exactly* match the motion of the output. When the motion requires more torque than the motor is able to provide, an error in output force appears, and this error is lower with low stiffness in the series elastic element. At the limit, an infinitely soft spring will show no force error for deflection of the output. In contrast, for the example of varying force on a stationary object, more optimal performance is achieved with a stiffer series elastic element, to the limit of optimal performance at infinite stiffness (complete rigidity). Any torque at the motor is directly and instantaneously transmitted to the output, without any deflection at all, such that inertia is not a factor in limiting the performance for this extreme example. (Note that for real systems with very high stiffness, sensing force becomes difficult and lag from loop delays creates higher errors in force (Kemper et al. 2010).) In both of these scenarios it is interesting that reducing reflected inertia increases performance. Only when reflected inertia is high, and significantly affects system dynamics, does series compliance become important for improving performance in certain applications.

**Series elastic actuators are made up of two fundamental components: a primary mover and an elastic element.** There are two ways to configure these two elements: with proximal elasticity or with distal elasticity, as shown in Fig. 3. Both configurations protect the gearing from large impacts and allow for high-fidelity force measurement by measuring spring deflection. The proximal elasticity case has an advantage that it allows for the sensor that measures the deformation of the elastic element to have less relative movement (Paine et al. 2014). The

disadvantage is that the physical mass/inertia (not the reflected inertia) of the motor is part of the unsprung output. Placing the elastic element on the distal side of the actuator allows for a more direct measure of the force applied to the output, without the motor mass in the way.

To show both a difference between proximal and distal series elasticity and the nuances of modeling series elastic actuators, consider two different linear actuator configurations shown in Fig. 3. In these diagrams,  $m_m$  is the mass of the motor and gearbox while  $m_o$  is the mass of the components on the output. Reflected inertia of the motor is described as a rack and pinion with pinion radius  $r$ , representing the speed reducer or gearbox and inertia  $I_m$ , representing the rotor inertia. This is intended to highlight that the inertia only exists with respect to relative movement between the two blocks, and does not add to the overall robot inertia (Spong 1987). The elastic element has stiffness  $k_s$  and damping  $b_s$ , while the motor has internal damping  $b_m$ . In the distal elasticity case, the mass of the motor is welded to the ground, and does not affect the dynamics of the actuator; it is shown in the diagram to highlight the difference between the reflected inertia and the motor mass.

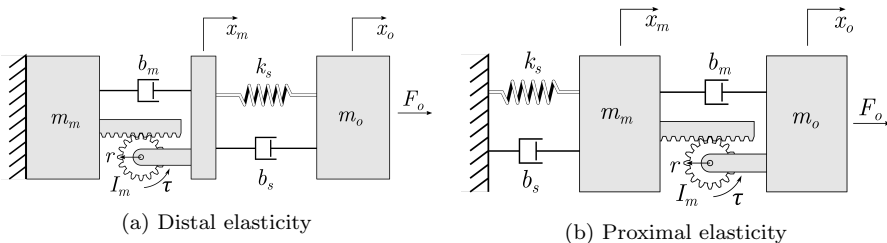


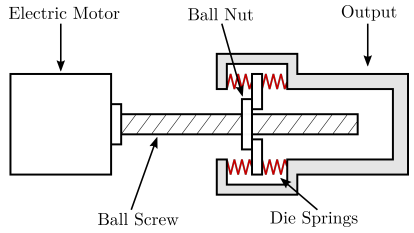
Fig. 3: Block diagrams of proximal and distal series elastic actuator configurations. Reflected inertia of the motor is represented by a rack and pinion.

To examine the difference between these two series elastic configurations and a rigid actuator, consider the case of an inelastic impact at the output. This impact could represent a robot arm contacting a rigid object or a robot leg impacting the ground. Let the output be moving with velocity  $\Delta v$  before the impact and be stationary immediately after the impact. The ratio of the impulse to the velocity change is an inertia which represents the total inertia of the system at the output. In the rigid case, this is simply motor's reflected inertia plus the inertia of the components on the output. By integrating the dynamics over the infinitesimal time of impact one can solve for the post impact velocities and required impulse. The expressions as well as a numeric evaluation using parameters from the MIT-SEA (one particular series elastic actuator implementation) are shown in Table 1 (Robinson 2000). The addition of either proximal or distal elasticity results in a much smaller impulse compared to the rigid case. Distal elasticity has the advantage that it decouples the output from the actuator reflected inertia more effectively.

**The implementation of a series elastic actuator translates the desired actuator dynamics into hardware.** Linear series elastic actuators can take the form of antagonistic die springs separating the motion source from the output.

Table 1: Inertia felt in an inelastic collision at the output of different configurations of a series elastic actuators. Both proximal and distal elasticity greatly reduce the magnitude of an impact.

No Elasticity	Distal Elasticity	Proximal Elasticity
$m_o + \frac{I_m}{r^2}$	$m_o$	$m_o + \frac{I_m}{r^2} \left( \frac{m_m}{m_m + I_m/r^2} \right)$
129 kg	0.261 kg	1.38 kg



(a) A generic linear series elastic actuator



(b) The MIT SEA (Robinson 2000)

Fig. 4: Linear series elastic actuators. Linear series elastic actuators generally utilize antagonistic die springs actuated through a ball screw.

An early example seen above in figure 4b is the MIT-SEA. This actuator was designed to act as a high-performance closed-loop force source. This requires the actuator have a soft enough spring to sense fine forces, but a stiff enough spring to allow for fast changes in the output force. The MIT-SEA is powered by a brushless DC motor that drives a ball screw to create the linear movement. A linear potentiometer measures the deflection of the springs (Robinson 2000).

In many robotic applications is it mechanically convenient to actuate a rotary joint for torque control. The general approach towards rotary series elastic actuators has been to use a gearbox to increase motor torque and couple the output through a stiff torsion spring for force sensing. Implementation examples include a cross shaped torsion bar (Williamson 1995), and a planar torsion spring. A planar torsion spring consisting of an inner and an outer ring connected by flexible splines was utilized in the Robonaut 2 robot (Diftler et al. 2011). Planar torsion springs hold an advantage in that they are much more compact compared to a torsion bar.

**Series Elasticity may also be used to store energy in an oscillating system.** The examples discussed so far use elasticity to regulate a generalized force or torque. In contrast, applications with the goal of energy storage, such as legged locomotion, dictate a different approach to implementing series elasticity. The springs must be physically much larger to store energy, while still being measurable; a successful approach in legged robots is to use a bending plate made from fiberglass or titanium (Grizzle et al. 2009; Brown and Zeglin 1998; Knabe et al. 2014). These materials allow for physically larger springs that can store much more energy per mass compared to steel die springs. As shown in Fig. 5, ATRIAS used

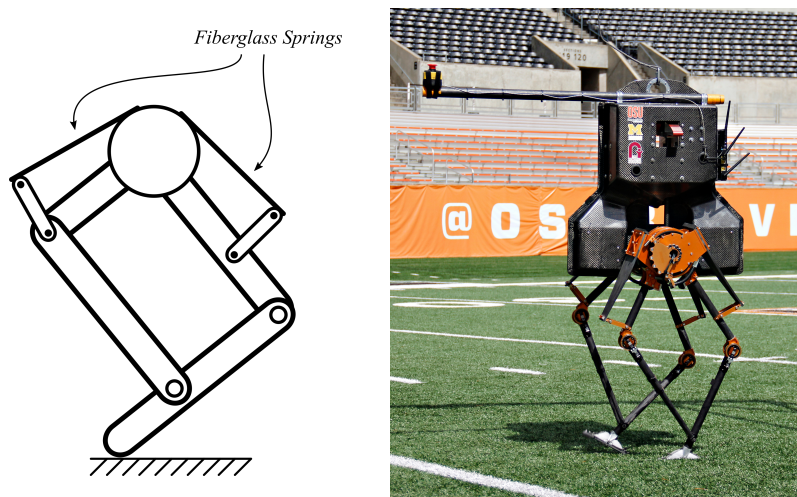


Fig. 5: The ATRIAS robot utilizes fiberglass springs to store energy and protect the gearboxes from impacts. The motors in the hip actuate the four bar mechanism through the fiberglass springs (Hubicki et al. 2016).

fiberglass springs to couple the gearbox output to the leg mechanism (Hubicki et al. 2016). In other robots, elastomers such as rubber provide larger specific energy densities than steel or composite springs can provide (Haldane et al. 2016). One of the difficulties when using an elastomer spring compared to a composite or steel spring is the intrinsic hysteresis and nonlinearity of elastomers (Rollinson et al. 2013).

**The amount of system integration versus system modularity of a series elastic actuator depends on the application.**

Series elastic actuators, as a modular unit, clearly provide some convenience in manufacturing and implementation. Each unit can have its own distributed closed loop force/torque controller, and a high level whole body controller can send joint force/torque commands to the individual joint controllers, assuming sufficient bandwidth for all desired tasks. A number of different modular actuator designs have been proposed in literature, along with control algorithms (Kong et al. 2009; Schutz et al. 2016; Paine et al. 2014; Pratt et al. 2004a). There are even commercial modular series elastic actuators available such as the ANYdrive (ANYbotics 2018), X-Series Actuator (HEBI Robotics 2018) and P170 Orion (Apptronik, inc. 2018). However, a single modular actuator design is not likely to be well suited for every joint in a robot, just as a single electric motor size is not well suited for every joint. For example, the wrist of a robot arm should have a softer spring and smaller peak torque compared to the shoulder joint.

An alternative approach is to design the individual actuators and passive elasticity with the goal of designing a system level compliance. For example, a robot arm may be designed to have a specific passive stiffness at the end effector or a robot leg may be designed to have a specific leg stiffness. Creating this type of behavior across a range of configurations may require designing nonlinearity into the springs or modifying the mechanism. Some examples of this approach being used





(a) Baxter, a collaborative robot used in industrial automation applications.



(b) A series elastic actuator powered ankle orthosis (Alam et al. 2014)

Fig. 6: Two robotic systems that utilize series elasticity for fine force control and intrinsic safety while interacting with people.

are the bow leg hopper, ScarLETH, and Cassie (Brown and Zeglin 1998; Hutter et al. 2011; Agility Robotics 2018).

#### **Applications: Elastic actuation is particularly useful in tasks that require physical interaction**

**Force control applications benefit from series elasticity.** In robot arms, sensitive force control and compliant interaction can allow for safe interaction with humans and can protect the robot from damaging itself during impacts. The use of series elastic actuators are one of the key factors that allowed for robot arms to transition from highly structured industrial applications to more generalized environments and interactions with people, such as the “Baxter” robot (figure 6a). Baxter is able to operate safely side by side with people principally because of the use of series elasticity in the joints. Series elasticity enables safe operation because of how it modifies the actuator dynamics. When a compliant robot arm contacts an unexpected object, such as a human, the elasticity allows it to sense an increase in torque at the joints. More importantly, the elasticity gives the robot time to decelerate its motors before the force at the end effector rises too high and causes damage (Bicchi et al. 2005).

When robots are designed specifically for physical human interaction, such as rehabilitation devices, elasticity and accurate force control are very important. For example, with assistive orthoses and prosthetics, low impedance force control is vital to user comfort. When connecting an actuator in parallel or series to a user’s joints, the reflected inertia of the motor and the reflected damping are very apparent (Pratt et al. 2004b). Series elastic actuators have been successfully utilized in many orthosis devices such as on arms, ankles and whole lower bodies (Ragonesi et al. 2011; Alam et al. 2014; Veneman et al. 2006).

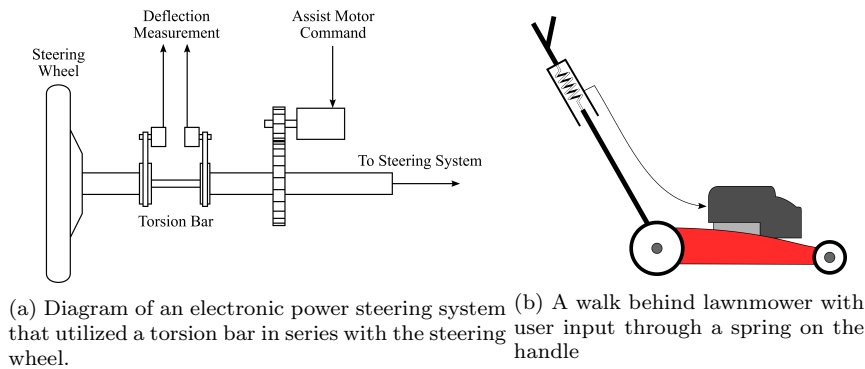


Fig. 7: Two examples of series elasticity that require interaction with a human operator.

The concept of series elastic actuation is used widely in systems that involve human interfaces, even if these systems are not commonly thought of as series elastic actuators. For example, one widespread use of these principles is the modern electronic power steering system in an automobile, shown in Fig. 7a. The steering column that connects the steering wheel to the steering system contains a torsion bar that is instrumented to measure the spring's deflection. This signal is used in the controller that commands the electric assist motor (Kim and Song 2002). Although the system does not attempt to control the steering wheel, the torsion bar and the assist motor work together to create a haptic (i.e. force) feedback system for the driver and amplify the driver's torque input.

Some self-powered lawnmowers can even be described as a series elastic actuator, as shown in Figure 7b. The operator pushes on a spring-loaded handle which opens the throttle on the engine. This increases the torque to the wheels, pushing the mower away from the person. The lawnmower engine and drive train is the actuator, the handle is the output and the connection between the spring loaded handle and the engine throttle is the control system. The lawnmower uses the engine to regulate a constant deflection in the spring, and maintain a near-constant force from the human operator to move the lawnmower, despite variations in the actuator work required to move the lawnmower forward.

**Legged robots benefit from series elasticity for reasons beyond force control.** The biomechanics of animals shows wide usage of and benefits from series elasticity. In animal legged locomotion, elasticity can store energy throughout a stride, improve swing leg energetics, and mitigate impacts with the ground (Alexander 1990). In a biological system, muscles (roughly) act as primary movers and tendons (roughly) act as elastic elements. In turkeys it was shown that tendons reduce metabolic cost in two primary ways: First, tendons store and release energy throughout a stride, lowering the amount of mechanical work the muscles need to provide. Second, tendons allow the muscles to contract less during a stride by providing much of the total deflection from their compliance, reducing the required power output from the muscle (same force, lower velocity movement) (Roberts 2002).

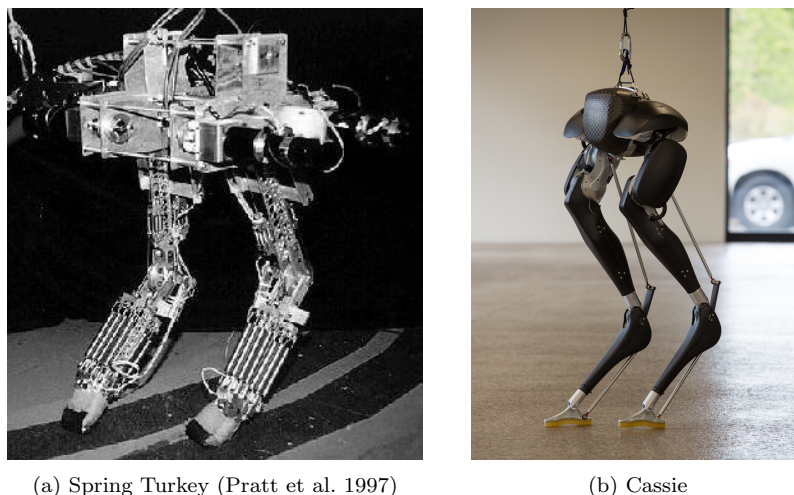


Fig. 8: Two legged robots that utilize series elasticity. Spring turkey uses elasticity for precise closed loop force control while Cassie uses elasticity to create excitable passive dynamics.

These benefits, apparent in animals, also apply to legged robots. A spring in series with the motor in a robot, properly tuned and controlled, reduces the power requirement on the motor throughout a stance phase, and mitigates large impact forces from the regular ground impacts. The ARL Monopod II, a planar hopping robot, utilized springs in both the leg extension actuator and the hip angle actuator to create passive dynamics that enable hopping. In fact, the robot was able to hop unactuated for a few steps before falling (Ahmadi and Buehler 1999). A similar approach more recently was used in Cassie, shown in Fig. 8b. Cassie is a 3D bipedal robot that has series elasticity built into the leg mechanism to create passive dynamics that enable efficient and robust locomotion.

Many legged robots utilize series elasticity to create a closed loop torque source, not to store energy. Pratt's early planar robots such as spring turkey (seen in Fig. 8a) and spring flamingo utilized this type of series elastic actuator. This allowed for the development of the whole body force control method known as virtual model control (Pratt et al. 1997; Pratt and Pratt 1998). Many subsequent legged robots also used this type of actuation strategy including the 3D robots: Valkyrie, M2V2, StarLETH and ANYmal (Montgomery et al. 2006; Pratt et al. 2012; Hutter et al. 2012, 2016).

**Future Directions for Research: Series elasticity is an important option in the engineers' bag of tools; will be relevant for nearly all physical interaction**

When designing robots, people generally focus on kinematics, degrees of freedom and accessible workspace. This can generally be described as a kinematic design process; but the space of dynamic behavior is at least as complex and influential to the overall performance. Designing with respect to the passive dynamics will

involve passive compliance, mass distribution, adjusting reflected inertia, and other effects. These effects must be applied to an entire system, and not just to individual joints, when considering system-level behaviors. Considering 2D or 3D compliance ellipse of an end-effector, which may be configuration-dependent, as well as inertial properties, are tools for dynamics-aware design (Park et al. 2009).

Utilization of series elasticity has clear benefits when trying to control forces. Elasticity is also critical for energy storage and power amplification in cyclic behaviors, such as walking and running gaits. Research is ongoing in this area, as roboticists push against limits of actuator dynamics, and improve them by reducing inertia and friction, or learn how best to implement series elasticity in multiple degrees of freedom, with varying stiffness functions that are configuration-dependent, to best support the behavior (Hobart et al. 2020; Schepelmann et al. 2014; Lakatos et al. 2014).

In developing complex compliance behaviors, research is also pushing towards a better understanding of the integration of control with the passive dynamics. Widespread whole body control methods such as operational space control, virtual model control or a general inverse dynamics do not account for underactuation due to elasticity in a comprehensive way, but extensions of these approaches or the addition of new tools such as machine learning may. Designing an entire system for a particular behavior, by utilizing series compliance, can become quite complex; more importantly, it must be a design process that is integrated with control and planning methods that treat a series elastic robots as a large, unified dynamical system (Remy 2011; Lakatos et al. 2017). Work in this direction will enable future robots that are capable of surprising, even human-like, performance.

## References

- Agility Robotics (2018) Agility robotics: Cassie. URL <http://www.agilityrobotics.com>
- Ahmadi M, Buehler M (1999) The arl monopod ii running robot: Control and energetics. In: *Robotics and Automation, 1999. Proceedings. 1999 IEEE International Conference on, IEEE*, vol 3, pp 1689–1694, DOI 10.1109/ROBOT.1999.770352, URL <http://ieeexplore.ieee.org/document/770352/>
- Alam M, Choudhury IA, Mamat AB (2014) Mechanism and design analysis of articulated ankle foot orthoses for drop-foot. *The Scientific World Journal* 2014
- Alexander R (1990) Three uses for springs in legged locomotion. *International Journal of Robotics Research* 9(2):53–61
- ANYbotics (2018) Anydrive. URL <https://www.anybotics.com/anydrive/>
- Appttronik, inc (2018) Appttronik p170 orion. URL <https://appttronik.com/product/appttronik-p170-orion/>
- Bicchi A, Tonietti G, Bavaro M, Piccigallo M (2005) Variable stiffness actuators for fast and safe motion control. In: *Robotics research. The eleventh international symposium*, Springer, pp 527–536
- Braun D, Howard M, Vijayakumar S (2012) Optimal variable stiffness control: formulation and application to explosive movement tasks. *Autonomous Robots* 33(3):237–253
- Brown B, Zeglin G (1998) The bow leg hopping robot. In: *Proceedings. 1998 IEEE International Conference on Robotics and Automation (Cat. No.98CH36146)*, IEEE, vol 1, pp 781–786, DOI 10.1109/ROBOT.1998.677072, URL <http://ieeexplore.ieee.org/document/677072/>
- De Schutter J (1987) A study of active compliant motion control methods for rigid manipulators based on a generic scheme. In: *Robotics and Automation. Proceedings. 1987 IEEE International Conference on, IEEE*, vol 4, pp 1060–1065
- Diftler M, Mehling J, Abdallah M, Radford N, Bridgwater L, Sanders A, Askew R, Linn D, Yamokoski J, Permenter F, Hargrave B, Platt R, Savely R, Ambrose R (2011) Robonaut 2 - The first humanoid robot in space. In: *2011 IEEE International Conference on*

- Robotics and Automation, IEEE, pp 2178–2183, DOI 10.1109/ICRA.2011.5979830, URL <http://ieeexplore.ieee.org/document/5979830/>
- Grizzle J, Hurst J, Morris B, Park HW, Sreenath K (2009) MABEL, a new robotic bipedal walker and runner. In: 2009 American Control Conference, IEEE, pp 2030–2036, DOI 10.1109/ACC.2009.5160550, URL <http://ieeexplore.ieee.org/document/5160550/>
- Haldane DW, Plecnik MM, Yim JK, Fearing RS (2016) Robotic vertical jumping agility via series-elastic power modulation. *Science Robotics* 1(1)
- HEBI Robotics (2018) X-series actuators. URL <https://www.hebirobotics.com/x-series-smart-actuators/>
- Hobart CG, Mazumdar A, Spencer SJ, Quigley M, Smith JP, Bertrand S, Pratt J, Kuehl M, Buerger SP (2020) Achieving versatile energy efficiency with the wanderer biped robot. *IEEE Transactions on Robotics* pp 1–8
- Hubicki C, Grimes J, Jones M, Renjewski D, Spröwitz A, Abate A, Hurst J (2016) ATRIAS: Design and validation of a tether-free 3D-capable spring-mass bipedal robot. *International Journal of Robotics Research* 35(12):1497–1521, DOI 10.1177/0278364916648388
- Hurst J, Rizzi A, Hobbelen D (2004) Series elastic actuation: Potential and pitfalls. In: International Conference on Climbing and Walking Robots, URL [http://mime.oregonstate.edu/research/drl/publications/\\_documents/hurst\\_2004a.pdf](http://mime.oregonstate.edu/research/drl/publications/_documents/hurst_2004a.pdf)
- Hutter M, Remy CD, Hoepflinger MA, Siegwart R (2011) ScarLETH: Design and control of a planar running robot. In: 2011 IEEE/RSJ International Conference on Intelligent Robots and Systems, IEEE, pp 562–567, DOI 10.1109/IROS.2011.6094504, URL <http://ieeexplore.ieee.org/document/6094504/>
- Hutter M, Gehring C, Bloesch M, Hoepflinger MA, Remy CD, Siegwart R (2012) STARLETH: A COMPLIANT QUADRUPEDAL ROBOT FOR FAST, EFFICIENT, AND VERSATILE LOCOMOTION. In: Adaptive Mobile Robotics, WORLD SCIENTIFIC, pp 483–490, DOI 10.1142/9789814415958\_0062, URL [http://www.worldscientific.com/doi/abs/10.1142/9789814415958\\_0062](http://www.worldscientific.com/doi/abs/10.1142/9789814415958_0062)
- Hutter M, Gehring C, Jud D, Lauber A, Bellicoso CD, Tsounis V, Hwangbo J, Bodie K, Fankhauser P, Bloesch M, Diethelm R, Bachmann S, Melzer A, Hoepflinger M (2016) ANYmal - a highly mobile and dynamic quadrupedal robot. In: 2016 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), IEEE, pp 38–44, DOI 10.1109/IROS.2016.7758092, URL <http://ieeexplore.ieee.org/document/7758092/>
- Kemper K, Koepl D, Hurst J (2010) Optimal passive dynamics for torque/force control. In: Robotics and Automation (ICRA), 2010 IEEE International Conference on, IEEE, pp 2149–2154
- Kim JH, Song JB (2002) Control logic for an electric power steering system using assist motor. *Mechatronics* 12(3):447–459, DOI 10.1016/S0957-4158(01)00004-6, URL <https://www.sciencedirect.com/science/article/pii/S0957415801000046>
- Knabe C, Lee B, Orekhov V, Hong D (2014) Design of a Compact, Lightweight, Electromechanical Linear Series Elastic Actuator. In: Volume 5B: 38th Mechanisms and Robotics Conference, ASME, p V05BT08A014, DOI 10.1115/DETC2014-35096, URL <http://proceedings.asmedigitalcollection.asme.org/proceeding.aspx?doi=10.1115/DETC2014-35096>
- Kong K, Bae J, Tomizuka M (2009) Control of rotary series elastic actuator for ideal force-mode actuation in human-robot interaction applications. *IEEE/ASME Transactions on Mechatronics* 14(1):105–118, DOI 10.1109/TMECH.2008.2004561, URL <http://ieeexplore.ieee.org/document/4783213/>
- Lakatos D, Rode C, Seyfarth A, Albu-Schäffer A (2014) Design and control of compliantly actuated bipedal running robots: Concepts to exploit natural system dynamics. In: 2014 IEEE-RAS International Conference on Humanoid Robots, pp 930–937
- Lakatos D, Friedl W, Albu-Schäffer A (2017) Eigenmodes of nonlinear dynamics: Definition, existence, and embodiment into legged robots with elastic elements. *IEEE Robotics and Automation Letters* 2(2):1062–1069
- Montgomery J, Roumeliotis SI, Johnson A, Matthies L (2006) Actuator Control for the NASA-JSC Valkyrie Humanoid Robot: A Decoupled Dynamics Approach for TorqueControl of Series Elastic Robots. *J Field Robotics* 23(3):245–267, DOI 10.1002/rob, 10.1.1.91.5767
- Paine N, Oh S, Sentis L (2014) Design and Control Considerations for High-Performance Series Elastic Actuators. *IEEE/ASME Transactions on Mechatronics* 19(3):1080–1091, DOI 10.1109/TMECH.2013.2270435, URL <http://ieeexplore.ieee.org/document/6555856/>

- Park JJ, Kim HS, Song JB (2009) Safe robot arm with safe joint mechanism using nonlinear spring system for collision safety. In: Robotics and Automation, 2009. ICRA'09. IEEE International Conference on, IEEE, pp 3371–3376
- Pratt G, Willisson P, Bolton C, Hofman A (2004a) Late motor processing in low-impedance robots: impedance control of series-elastic actuators. American Control Conference, 2004 Proceedings of the 2004 4:3245–3251 vol.4, DOI 10.1109/ACC.2004.182786
- Pratt J, Dilworth P, Pratt G (1997) Virtual model control of a bipedal walking robot. 997 IEEE International Conference on Robotics and Automation, 1997 Proceedings, 1 (April):193–198, DOI 10.1109/ROBOT.1997.620037
- Pratt J, Koolen T, de Boer T, Rebula J, Cotton S, Carff J, Johnson M, Neuhaus P (2012) Capturability-based analysis and control of legged locomotion, Part 2: Application to M2V2, a lower-body humanoid. The International Journal of Robotics Research 31(10):1117–1133, DOI 10.1177/0278364912452762, URL <http://journals.sagepub.com/doi/10.1177/0278364912452762>
- Pratt JE, Pratt GA (1998) Exploiting natural dynamics in the control of a planar bipedal walking robot. In: Proceedings of the Annual Allerton Conference on Communication Control and Computing, Citeseer, vol 36, pp 739–748, DOI 10.1007/BFb0035216
- Pratt JE, Krupp BT, Morse CJ, Collins SH (2004b) The roboknee: an exoskeleton for enhancing strength and endurance during walking. In: Robotics and Automation, 2004. Proceedings. ICRA'04. 2004 IEEE International Conference on, IEEE, vol 3, pp 2430–2435
- Ragonese D, Agrawal S, Sample W, Rahman T (2011) Series elastic actuator control of a powered exoskeleton. In: 2011 Annual International Conference of the IEEE Engineering in Medicine and Biology Society, IEEE, pp 3515–3518, DOI 10.1109/IEMBS.2011.6090583, URL <http://ieeexplore.ieee.org/document/6090583/>
- Remy CD (2011) Optimal Exploitation of Natural Dynamics in Legged Locomotion. PhD Dissertation, ETH Zurich (19831), DOI 10.3929/ethz-a-6665065
- Roberts TJ (2002) The integrated function of muscles and tendons during locomotion. Comparative Biochemistry and Physiology Part A: Molecular & Integrative Physiology 133(4):1087–1099
- Robinson David W (David William) (2000) Design and analysis of series elasticity in closed-loop actuator force control. PhD thesis, Massachusetts Institute of Technology, URL <https://dspace.mit.edu/handle/1721.1/54838#files-area>
- Rollinson D, Ford S, Brown B, Choset H (2013) Design and modeling of a series elastic element for snake robots. In: ASME 2013 Dynamic Systems and Control Conference, American Society of Mechanical Engineers, pp V001T08A002—V001T08A002
- Schepelmann A, Geberth KA, Geyer H (2014) Compact nonlinear springs with user defined torque-deflection profiles for series elastic actuators. In: 2014 IEEE International Conference on Robotics and Automation (ICRA), pp 3411–3416
- Schutz S, Mianowski K, Kottling C, Nejadfard A, Reichardt M, Berns K (2016) RRLAB SEA - A highly integrated compliant actuator with minimised reflected inertia. IEEE/ASME International Conference on Advanced Intelligent Mechatronics, AIM 2016-Sept:252–257, DOI 10.1109/AIM.2016.7576775
- Spong MW (1987) Modeling and Control of Elastic Joint Robots. Journal of Dynamic Systems, Measurement, and Control 109(4):310–318, DOI 10.1115/1.3143860, URL <https://doi.org/10.1115/1.3143860>, <https://asmedigitalcollection.asme.org/dynamicsystems/article-pdf/109/4/310/5604812/310.1.pdf>
- Veneman JF, Ekkelenkamp R, Kruidhof R, van der Helm FC, van der Kooij H (2006) A Series Elastic- and Bowden-Cable-Based Actuation System for Use as Torque Actuator in Exoskeleton-Type Robots. The International Journal of Robotics Research 25(3):261–281, DOI 10.1177/0278364906063829, URL <http://journals.sagepub.com/doi/10.1177/0278364906063829>
- Williamson MM (1995) Series Elastic Actuators. PhD thesis, Massachusetts Institute of Technology, URL <https://dspace.mit.edu/handle/1721.1/6776#files-area>

**Cross-References**

Haptics and Haptic Interfaces, Variable impedance actuators, Force Control, Impedance Control