RUNNING ON SOFT GROUND: SIMPLE, ENERGY-OPTIMAL DISTURBANCE REJECTION*

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Soft ground is a ubiquitous hazard for legged locomotion and has yet to be conquered in a robust, dynamic, and economical manner. In search of a controller to meet these demands, we found that a simple force controller is energy optimal for spring-loaded running on unknown ground dissipation. The simplicity of this optimal controller suggests a fundamental insight into legged locomotion.

Keywords: legged, running, terrain, optimal control, force control, SLIP

1. Introduction

Handling uncertain terrain is an enduring challenge in legged locomotion, especially when running. We believe the solution lies in fundamental principles of spring-mass running. Some controllers for the Spring-Loaded Inverted Pendulum (SLIP) have been successful in rejecting potholes and slopes by adjusting leg posture mid-flight. However, negotiating soft terrain, such as soil or sand, requires thoughtful actuation while in brief contact with the ground. This challenge is further complicated as the SLIP model is nonlinear, hybrid-dynamical, high-dimensional, underactuated, and analytically unsolvable. This investigation discovered a simple feedback force controller that completely rejects soft ground in a single step for minimum energy cost.

In simulation, we demonstrate that the controller navigates even extremely soft ground without any estimates or foreknowledge of the ground

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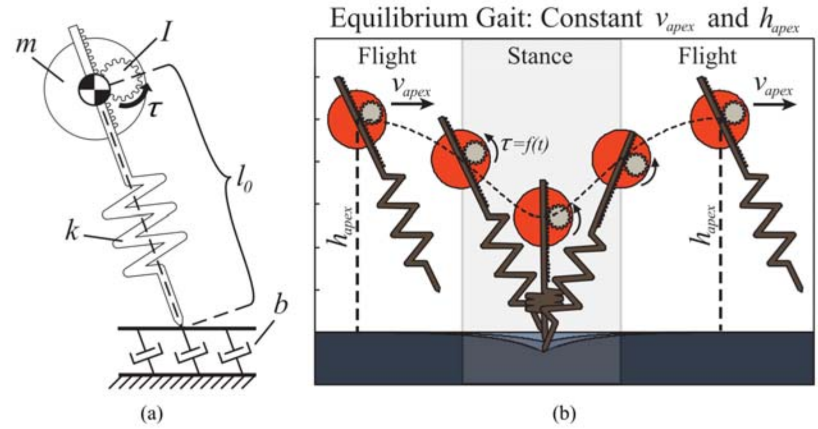

Fig. 1: (a) An actuated variant of the Spring-Loaded Inverted Pendulum (SLIP) model, including a motor that regulates axial spring compression during stance, and a linear damping ground model acting inline with the leg. (b) A visualization of an equilibrium gait, where apex velocity and height remain constant across flight phases. On soft ground, this is only possible if the actuator intervenes during the stance phase, as shown above.

properties. Further, we present evidence from numerical optimizations that this controller is, in fact, the energy-optimal solution for handling any dissipative terrain. We conclude that these features make the force controller well-suited for real-world locomotion and realizable on hardware that approximates SLIP-model running.

Solving an optimal control problem with such mathematical barriers required an unconventional approach. We use trajectory optimization as a starting point, manually inspecting numerical optimal trajectories for similarities. Informed by these similarities, we hypothesized a general optimal controller and, as validation, compared it to the performance of optimal trajectories. We call this process *optimization-inspired* controller synthesis.

2. Background

The research trail of running robot control was blazed by Raibert,[1] whose simple and effective controllers inspired a wave of stable, energy-based running controllers.[2] Hybrid-Zero Dynamics, while having largely been applied to walking,[3] has recently yielded robust and efficient running gaits[4] on the spring-legged robot, MABEL. For the idealized, energetically-conservative

SLIP model, Ernst, Geyer, and Blickhan[5] (EGB) developed a feedforward steady-state flight-phase controller. While airborne, EGB control selects a leg angle which guarantees the same flight speed on the next step (an *equilibrium gait*) even without detecting the impending ground. However, guarantees made by EGB control and its predecessors assume a rigid landing surface. This investigation seeks a stance-phase controller complementary to EGB control which rejects the effect of soft surfaces while performing minimal actuator work. Such a controller, as Fig. 1b illustrates, would ensure economical, robust control while running on surfaces like soil, sand, and snow.

3. Methods

3.1. *Trajectory Optimization*

Using the actuated SLIP model shown in Fig. 1a, a trajectory optimization problem was formulated. Given **A)** an initial apex condition, **B)** a leg angle at touch down[a], and **C)** a ground-damping coefficient, find a torque trajectory[b] which **1)** results in an equilibrium gait[c] (as defined in Fig. 1b), and **2)** minimizes actuator work[d].

Using Sequential Quadratic Programming[e], energy-optimal torque trajectories were computed for the SLIP model[f] at various initial conditions and assorted surface consistencies. However, any individual trajectory is only useful for its respective narrow case and, at 10^3 seconds per optimization, are cripplingly complicated to compute in real time. While these trajectories are impractical on their own, we suspected that inspecting them all for similarities could yield insight into a simple, general solution.

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[a] On rigid ground, these leg angles at touch down would produce equilibrium gaits, and are computed using EGB control.

[b] The input trajectory is a piecewise-linear, time-scheduled, torque applied to the leg-length control motor at the moment of touch down.

[c] An "equilibrium gait", similar to a "limit-cycle" gait, "periodic gait", or "steady-state gait", has equivalent state variables from step to step at the apex of flight.

[d] The objective function is defined as the time integral of the unsigned derivative of actuator work, resulting in minimal motor intervention in the passive dynamics.

[e] Using `MATLAB`'s constrained minimization algorithm, `fmincon`, with an equality constraint.

[f] SLIP model parameters are estimates from our experimental monopod, ATRIAS 2.0: $m = 32kg$, $l_0 = 0.90m$, $k = 3.0\frac{kN}{m}$, $\tau_{max} = 25Nm$, motor inertia $I = 0.003kgm^2$, transmission ratio: 50:1, and ground damping coefficient, b, is experiment dependent.

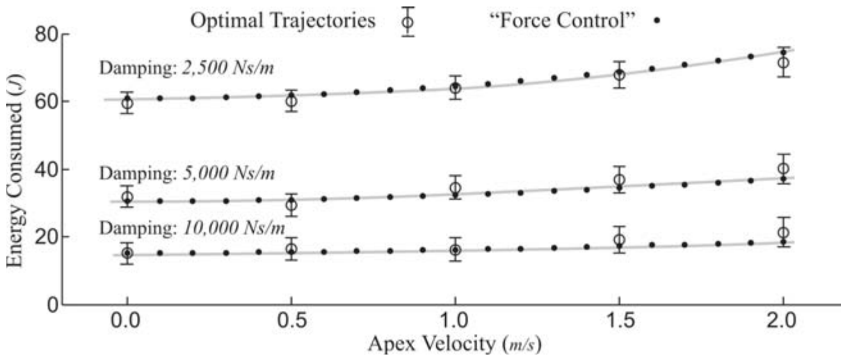

Fig. 2: Energy consumption of "force control" matches optimized trajectories, strongly suggesting that force control is an optimal solution to handling soft terrain. Energy error bars correspond to a 1% solver constraint tolerance.

3.2. *General Optimal Control*

Despite the optimal torque trajectories being widely varied, the axial ground reaction forces were remarkably similar across all tested surfaces. Even when a particularly soft surface caused the robot to sink 10% of its leg into the ground, conditions which could be likened to a sand dune or snow drift, the axial forces were nearly identical to that of rigid ground. This surprising similarity among energy-optimal trajectories led us to formulate a hypothesis: *"a controller that replicates the axial forces experienced on rigid ground is an energy-optimal control policy on soft ground."*

To test this hypothesis, we compared the energy economy of an axial-force controller to optimal energy consumption. To facilitate the necessary closed-loop control of the ground-reaction forces, a PI controller[g] was implemented which tracks the axial force trajectory experienced on rigid ground (dubbed *force control* for brevity).

4. Results and Conclusions

We compared the energy consumption and disturbance rejection of both force control and optimal trajectories. As shown in Fig. 2, when applied to the same situations as the optimization, force control preserved the equilibrium gait[h] on soft ground for the same energy cost as the optimized

[g]Integral control was necessary to eliminate steady-state error; Gains of $K_P = 10m$, $K_I = 20\frac{m}{s}$ were manually tuned.
[h]Equilibrium gaits were reliably produced within a 1% state variable tolerance.

trajectories[i]. We present this as numerical evidence which confirms our hypothesis that *force control is the energetically optimal solution to running on soft ground*.

We believe this solution is a fundamental insight into control of the SLIP model. By using numerical optimizations to inspire and test hypotheses about optimal control, we were able to bypass the mathematical complexity inherent to the model. We believe this approach, coined *optimization-inspired* controller synthesis, can apply to a broad class of problems.

Force control also has many practical advantages. The policy is simple, tracking easily computed force profiles instead of expensively computed optimal input trajectories. Further, it requires no estimation of the ground dynamics in order to be robust or optimal. This feature combines well with EGB flight control, since EGB requires no knowledge of the ground geometry, as demonstrated in a recent simulation study.[6] These features allow force control to be tested on practical surfaces with robotic hardware that can approximate SLIP-model dynamics.

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[i]Small variance in the SQP performance is likely due to local minima caused by trajectory representation and the state variable tolerance allowing slightly "lazier" solutions.